Equations for Part-of-Speech Tagging

Eugene Charniak and Curtis Hendrickson and Neil Jacobson and Mike Perkowitz
Department of Computer Science
Brown University
Providence RI 02912

Abstract
We derive from first principles the basic equations for a few of the basic hidden-Markov-model word taggers as well as equations for other models which may be novel (the descriptions in previous papers being too sparse to be sure). We give performance results for all of the models. The results from our best model (96.45% on an unused test sample from the Brown corpus with 181 distinct tags) is on the upper edge of reported results. We also hope these results clear up some confusion in the literature about the best equations to use. However, the major purpose of this paper is to show how the equations for a variety of models may be derived and thus encourage future authors to give the equations for their model and the derivations thereof.

Introduction
The last few years have seen a fair number of papers on part-of-speech tagging — assigning the correct part of speech to each word in a text [1,2,4,5,7,8,9,10]. Most of these systems view the text as having been produced by a hidden Markov model (HMM), so that the tagging problem can be viewed as one of deciding which states the Markov process went through during its generation of the text. (For an example of a system which does not take this view, see [2].) Unfortunately, despite the obvious mathematical formulation that HMM’s provide, few of the papers bother to define the mathematical model they use. In one case this has resulted in a confusion which we address subsequently. In most every case it has meant that large parts of the models are never described at all, and even when they are described the English descriptions are often vague and the occasional mathematical symbol hard to interpret as one is lacking a derivation of the equations in which it should rest.

In this paper we hope to rectify this situation by showing how a variety of Markov tagging models can be derived from first principles. Furthermore, we have implemented these models and give their performance. We do not claim that any of the models perform better than taggers reported elsewhere, although the best of them at 96.45% is at the upper end of reported results. However, the best taggers all perform at about the same level of accuracy. Rather our goal is to systematize the “seat of the pants” knowledge which the community has already accumulated. One place where we might be breaking new ground is in techniques for handling the sparse-data problems which inevitably arise. But even here it is hard to be sure if our techniques are new since previous authors have barely mentioned their sparse-data techniques, much less formalized them. We believe that providing a clean mathematical notation for expressing the relevant techniques will take this area out of the realm of the unmentionable and into that of polite scientific discussion.

The Simplest Model
We assume that our language has some fixed vocabulary, \( \{w^1, w^2, \ldots, w^n\} \). This is a set of words, e.g., \( \{a, and, vark, \ldots, zygote\} \). We also assume a fixed set of parts of speech, or tags, \( \{t^1, t^2, \ldots, t^2\} \), e.g., \( \{\text{adjective, adverb, \ldots, verb}\} \). We consider a text of \( n \) words to be a sequence of random variables \( W_{1,n} = W_1 W_2 \ldots W_n \). Each of these random variables can take as its value any of the possible words in our vocabulary. More formally, let the function \( V(X) \) denote the possible values (outcomes) for the random variable \( X \). Then \( V(W_i) = \{w^1, w^2, \ldots, w^n\} \). We denote the value of \( W_1 \) by \( w_1 \), and a particular sequence of \( n \) values for \( W_{1,n} \) by \( w_{1,n} \). In a similar way, we consider the tags for these words to be a sequence of random variables \( T_{1,n} = T_1 T_2 \ldots T_n \). A particular sequence of values for these is denoted as \( t_{1,n} \), and the \( i \)th one of these is \( t_i \). The tagging problem can then be formally defined as finding the sequence of tags \( t_{1,n} \) which is the result of the following function:

\[
T(w_{1,n}) \overset{\text{def}}{=} \arg \max_{t_{1,n} \in V(T_{1,n})} P(T_{1,n} = t_{1,n} \mid W_{1,n} = w_{1,n})
\]

In the normal way we typically omit reference to the random variables themselves and just mention their values. In this way Equation 1 becomes:

\[
T(w_{1,n}) = \arg \max_{t_{1,n}} P(t_{1,n} \mid w_{1,n})
\]
We now turn Equation 2 into a more convenient form.

\[
T(w_{1,n}) = \arg\max_{t_{1,n}} \frac{P(t_{1,n}, w_{1,n})}{P(w_{1,n})} \tag{3}
\]

\[
= \arg\max_{t_{1,n}} P(t_{1,n}, w_{1,n}). \tag{4}
\]

In going from Equation 3 to 4 we dropped \(P(w_{1,n})\) as it is constant for all \(t_{1,n}\).

Next we want to break Equation 4 into “bite-size” pieces about which we can collect statistics. To a first approximation there are two ways this can be done. The first is like this:

\[
P(t_{1,n}, w_{1,n}) = P(w_{1}) P(t_{1} \mid w_{1}) P(w_{2} \mid t_{1}, w_{1}) \]

\[
= P(w_{1}) P(t_{1} \mid w_{1}) \prod_{i=2}^{n} P(w_{i} \mid t_{1,i-1}, w_{1,i-1}) \]

\[
= \prod_{i=1}^{n} P(w_{i} \mid t_{i,i-1}, w_{1,i-1}) \]

\[
P(t_{i} \mid t_{i,i-1}, w_{1,i}) \] \(\tag{5}
\]

Here we simplified Equation 6 to get Equation 7 by suitably defining terms like \(t_{1,0}\) and their probabilities. We derived Equation 7 by first breaking out \(P(w_{1})\) from \(P(t_{1,n}, w_{1,n})\). In a similar way we can first break out \(P(t_{i})\), giving this:

\[
P(t_{1,n}, w_{1,n}) = \prod_{i=1}^{n} P(t_{i} \mid t_{1,i-1}, w_{1,i-1}) \]

\[
= \prod_{i=1}^{n} P(w_{i} \mid t_{i,i-1}, w_{1,i-1}) \]

\[
P(t_{i} \mid t_{1,i-1}, w_{1,i}) \] \(\tag{6}
\]

Substituting these equations into Equation 7, and substituting that into Equation 4 we get:

\[
T(w_{1,n}) = \arg\max_{t_{1,n}} \prod_{i=1}^{n} P(w_{i} \mid w_{1,i-1}) \]

\[
P(t_{i} \mid w_{i}) \] \(\tag{7}
\]

Equation 12 has a very simple interpretation. For each word we pick the tag which is most common for that word. This is our simplest model.

**Estimation of Parameters**

Before one can use such a model however, one still needs to estimate the relevant parameters. For Equation 12 we need the probabilities of each possible tag for each possible word: \(P(t^{i} \mid w^{j})\). The most obvious way to get these is from a corpus which has been tagged by hand. Fortunately there is such a corpus, the Brown Corpus [6] and all of the statistical data we collect are from a subset of this corpus consisting of 90% of the sentences chosen at random. (The other 10% we reserve for testing our models.) So, let \(C(t^{i}, w^{j})\) be the number of times the word \(w^{j}\) appears in our training corpus with the tag \(t^{i}\). In the obvious way \(C(w^{j}) = \sum_{t} C(t^{i}, w^{j})\). Then one approximation to the statistics needed for Equation 12 is the following estimate:

\[
P(t^{i} \mid w^{j}) \approx \frac{C(t^{i}, w^{j})}{C(w^{j})}. \tag{8}
\]

However, Equation 13 has problems when the training data is not complete. For example, suppose there is no occurrence of word \(w^{w}\). First, the quotient in Equation 13 is undefined. As this is a problem throughout this paper we henceforth define zero divided by zero to be zero. But this still means that \(P(t^{i} \mid w^{w})\) is zero for all \(t^{i}\).

We solve this problem by adding further terms to Equation 13. We model this after what is typically done in smoothing tri-gram models for English [7]. Thus we add a second term to the equation with weights attached to each term saying how heavily that term should be counted. That is, we are looking for an equation of the following form:

\[
P(t^{i} \mid w^{j}) \approx \lambda_{1}(w^{j}) \frac{C(t^{i}, w^{j})}{C(w^{j})} + \lambda_{2}(w^{j}) f(t^{i}, w^{j}). \tag{9}
\]

Here \(f(t^{i}, w^{j})\) is standing in for our as yet to be disclosed improvement. The two \(\lambda_{s}\) are the weights to be given to each term. Note that they can be different for different \(w\) and thus we have made them functions of \(w^{j}\). For any \(w\) Equation 14 must sum to one over all \(t^{i}\). We ensure this by requiring that the \(\lambda_{s}\) sum to one and that the terms they combine do so as well.
If we are primarily concerned about estimating $P(t^i \mid w^j)$ for $w^j$ which have not been encountered before the $\lambda$s can take a particularly simple form:

$$
\lambda_1(w^j) = \begin{cases} 
1 & \text{if } C(w^j) \geq 1 \\
0 & \text{otherwise}.
\end{cases}
$$

(15)

With these $\lambda$s the second term of Equation 14 should be the probability that a token of a word $w^j$ which we have never seen before has the tag $t^i$. Obviously we cannot really collect statistics on something that has never occurred, however when we were gathering our count data in the first place we often encountered words which up to that point had not been seen. We collect statistics on these situations to stand in for those which occur in the test data. Thus, let $C_p(t^i)$ be the number of times a word which has never been seen with the tag $t^i$ get this tag, and let $C_n()$ be the number of such occurrences in total. Then our improved probability estimation equation is this:

$$
P(t^i \mid w^j) \approx \lambda_1(w^j) \frac{C(t^i, w^j)}{C(w^j)} + \lambda_2(w^j) \frac{C_p(t^i)}{C_n()} \quad (16)
$$

With this improved parameter estimation function we are now able to collect statistics from our corpus and test the model thereby derived on our test corpus. The results are quite impressive for so simple a model: 90.25% of the words in the test data are labeled correctly. (The data for all of the models is summarized at the end of the paper in Figure 2.)

The “Standard” Model

The model of Equations 12 and 16 does not take any context into account. It simply chooses the most likely tag for each word out of context. Next we develop a model which does take context into account.

This time we start from Equation 8 and simplify it by making the following two Markov assumptions:

$$
P(t_i \mid t_{i-1}, w_{1:i-1}) = P(t_i \mid t_{i-1}) \quad (17)
$$

$$
P(w_i \mid t_{1:i}, w_{1:i-1}) = P(w_i \mid t_{i}) \quad (18)
$$

That is, we assume that the current tag is independent of the previous words and only dependent on the previous tag. Similarly we assume that the correct word is independent of everything except knowledge of its tag. With these assumptions we get the following equation:

$$
T(w_{1:n}) = \arg\max_{t_{1:n}} \prod_{i=1}^{n} P(t_i \mid t_{i-1}) P(w_i \mid t_i) \quad (19)
$$

A more important difference is that many do not use Equation 19 or the just-mentioned variants, but rather:

$$
T(w_{1:n}) = \arg\max_{t_{1:n}} \prod_{i=1}^{n} P(t_i \mid t_{i-1}) P(t_i \mid w_i) \quad (20)
$$

The difference is in the last term. This equation is found in [4,9] and is described in words in [5]. (However, while Church gives Equation 20 in [4], the results cited there were based upon Equation 19 (Church, personal communication).)

Equation 20 seems plausible except that it is virtually impossible to derive it from basic considerations (at least we have been unable to do so). Nevertheless, given the drastic Markov assumptions we made in the derivation of Equation 19 it is hard to be sure that its comparative theoretical purity translates into better performance. Indeed, the one paper we are acquainted with in which the comparison was made [1] found that the less pure Equation 20 gave the better performance. However, this was on a very small amount of training data, and thus the results may not be accurate.

To determine which, in fact, does work better we trained both on 90% of the Brown Corpus and tested on the remainder. We smoothed the probabilities using Equation 16. To use this on Equation 19 we made the following change in its form:

$$
T(w_{1:n}) = \arg\max_{t_{1:n}} \prod_{i=1}^{n} P(t_i \mid t_{i-1}) \frac{P(w_i) P(t_i \mid w_i)}{P(t_i)} \quad (21)
$$

$$
= \arg\max_{t_{1:n}} \prod_{i=1}^{n} P(t_i \mid t_{i-1}) \frac{P(t_i \mid w_i)}{P(t_i)} \quad (22)
$$

It was also necessary to smooth $P(t_i \mid t_{i-1})$. In particular we found in our test data consecutive words with unambiguous tags, where the tags had not been seen consecutively in the training data. To overcome this problem in the simplest fashion we smoothed the probability as follows:

$$
P(t_i \mid t_{i-1}) \approx (1 - \epsilon) \frac{C(t_{i-1}, t_i)}{C(t_{i-1})} + \epsilon. \quad (23)
$$

Here $\epsilon$ is a very small number so that its contribution is swamped by the count data unless that contributes zero. The net effect is that when there is no data on tag context the decision is made on the basis of $P(w^j \mid t^i)$ in Equation 19 or $P(t^i \mid w^j)$ in Equation 20.

The results were unequivocal. For Equation 19 we got 95.15% correct while for the less pure Equation 20 the results were poorer, 94.69%. While this may not seem like a huge difference, a better way to think of it is that we got an 18% reduction in errors. Furthermore, given that the models have exactly the same complexity, there is no cost for this improvement.
**An Improved Smoothing Model**

The smoothing model of Equation 16 is very crude. This and the subsequent section improve upon it. One problem is that the model uses raw counts to estimate the probabilities for a word’s tags once it has seen a word, even if only once. Obviously, if we have seen a word, say, 100 times, the counts probably give a good estimate, but for words we have seen only once they can be quite inaccurate. The improvement in this section is concerned with this problem.

In Equation 16 we collected statistics on the first occurrences of words we saw in the training data and used these statistics to predict what would happen on the first occurrences of words we saw in the test data. To improve our model we try to estimate the probability that we will next see the tag $t^i$ as the tag for $w^j$ despite the fact that it has never appeared as the tag for $w^j$ before

$$P(t^i \mid w^j) \text{ is to use extra training data to train the HMM corresponding to Equations 22 and 26 to find a (locally) best set of } \lambda \text{-values as done in [7]. However, it is possible to provide an argument for what their values ought to be. If we think about an HMM for producing the part of speech given the word, for each word there would be arcs leaving the state corresponding to each possible tag. (In fact, there would be several arcs for each tag, each going to a different next state, but we ignore this.) This would appear as shown on the upper portion of Figure 1. We assume that for $w^j$ we have seen $v$ of the tags, $t^1$ to $t^v$, and the rest, $t^{v+1}$ to $t^T$, have not been observed for $w^j$. The idea of Equation 26 is that the first term estimates the probabilities associated with the arcs $t^1$ to $t^v$, and the second term estimates the rest. To make the HMM look more like this equation we can transform it into the form on the lower portion of Figure 1 where we have introduced two $\epsilon$ (no output) transitions with probabilities $\lambda_1(w^j)$ and $\lambda_2(w^j)$, respectively. From this HMM it is easier to see the significance of these two probabilities, $\lambda_1(w^j)$ is the probability that the next occurrence of $w^j$ has as an associated tag, a tag which has occurred with $w^j$ before, and $\lambda_2(w^j)$ is the probability that it is a new tag for $w^j$. This latter term is the sum in the denominator of the second term of Equation 26.

**Morphological Help for Tagging**

The second improvement to our smoothing function uses the word-ending information to help determine correct tags. For example, if we have never seen the word “ rakishly, ” then knowledge that “ly” typically ends an adverb will improve our accuracy on this word — similarly, for “randomizing.”

We do not want to tackle the problem of determining the morphology of English words in this paper. Rather we assume that we have available a program which assigns roots and suffixes (we do not deal with any other kind of morphological feature) to our corpus and does so...
as well for those words in our test corpus which have appeared in the training corpus. For the words in the test corpus which have not appeared in the training corpus the morphological analyzer produces all possible analyses for that word and part of the problem we face is deciding between them. We should note that the morphological analyzer we had was quite crude and prone to mistakes. A better one would no doubt improve our results.

To accommodate our morphological analysis we now consider probabilities for different root-suffix combinations. Following our earlier conventions, we have a set of roots \(\{r^1, \ldots, r^\rho\}\) and a set of suffixes \(\{s^1, \ldots, s^\sigma\}\). \(r_{1,n}\) and \(s_{1,n}\) are sequences of \(n\) roots and suffixes with \(r_i\) and \(s_i\) being the \(i\)th one of each.

The probability that a word \(w_{1,n}\) is formed from a root-suffix sequence \((r_{1,n}, s_{1,n})\) is given by:

\[
\mathcal{T}(w_{1,n}) = \arg \max_{t_{1,n}, r_{1,n}, s_{1,n}} \sum_{t_{1,n}} P(t_{1,n}, r_{1,n}, s_{1,n} \mid w_{1,n})
\]

This can be written more explicitly as:

\[
\mathcal{T}(w_{1,n}) = \arg \max_{t_{1,n}, r_{1,n}, s_{1,n}} \frac{P(t_{1,n}, r_{1,n}, s_{1,n}, w_{1,n})}{P(w_{1,n})}
\]

We delete \(w_{1,n}\) in going from Equation 28 to 29 because roots plus suffixes determine the words. However, this means that in all of the equations after that point there is an implicit assumption that we are only considering root as suffixes which combine to form the desired word.

Next we make some Markov assumptions.

\[
P(t_i \mid t_{i-1}, r_{i-1}, s_{i-1} = 1) = P(t_i \mid t_{i-1})
\]

\[
P(r_i, s_i \mid t_{i}, r_{i-1}, s_{i-1} = 1) = P(r_i, s_i \mid t_{i})
\]

\[
P(r_i, s_i \mid t_i) = P(r_i \mid t_i) P(s_i \mid t_i)
\]

With these assumptions we can manipulate Equation 31 as follows:

\[
\mathcal{T}(w_{1,n}) = \arg \max_{t_{1,n}, r_{1,n}, s_{1,n}} \sum_{t_{1,n}} \prod_{i=1}^{n} P(t_i \mid t_{i-1}) P(s_i \mid t_i) P(r_i) \frac{P(t_i \mid r_i)}{P(t_i)}
\]

Equation 36 is a version of Equation 19, but adapted to morphological analysis. It differs from the earlier equation in three ways. First, it includes a new term, \(P(s_i \mid t_i)\), for which we now need to gather statistics. However, since the number of tags and suffixes are small, this should not provide any difficult sparse-data problem. Second, rather than needing to smooth \(P(t_i \mid r_i)\) as in Equation 19, we now need to smooth \(P(t_i \mid r_i)\). However, it seems reasonable to continue to use Equation 26, with \(r_i\) substituted for \(w_i\). Finally, there is the term \(P(r_i)\), and this deserves some discussion.

There was no term corresponding to \(P(r_i)\) in Equation 19 as the term which would have corresponded to it was \(P(w_i)\), and that was removed as it was the same for all tags. However, in Equation 36 we are summing over roots, so \(P(r_i)\) is not a constant. In particular assuming that we would want our program to interpret some new word, e.g., “rakhishly” as “rakish” + “ly,” (or even better, “rake” + “ish” + “ly,” if our morphological analyzer could handle it) it would be the \(P(r_i)\) term which would encourage such a preference in Equation 36. It would do so because the probability of the shorter root would be much higher than the longer ones.

To model \(P(r_i)\) we have adopted a spelling model along the lines of the one used for the spelling of unknown words in [3]. This combines a Poisson distribution over word lengths with a maximum at 5, times a distribution over letters. We adopted a unigram model for letters. Here \(|r_i|\) is the length of \(r_i\) and \(l_i\) is the \(i\)th letter of \(r_i\).

\[
P(r_i) \equiv \delta_{|r_i|} \sum_{j=1}^{5} \sum_{k=1}^{|r_i|} \frac{1}{l_i} P(l_i \mid l_{i-1})
\]

**Results**

The results of our experiments are summarized in Figure 2. We trained our models on the Brown corpus with every tenth sentence removed (starting with sentence 1) and tested on these removed sentences. There were 114263 words in the test corpus. For the more basic methods we did experiments on both the full Brown corpus tag set (471 different tags) and a reduced set (186 tags). Most of the tags in the full set are “complex” tags that they consist of a basic tag plus one or more
tag modifiers. For those familiar with the Brown Corpus, to get the reduced set we stripped off the modifiers “FW” (foreign word), “TL” (title), “NC” (cited word), and “HL” (headline). For the more complex techniques we only used the reduced set since the basic dynamic programming algorithm for finding the best tags runs in big-O time \( \tau^2 \) where \( \tau \) is the number of different tags. Using normal tests for statistical significance we find that for the interesting cases of Figure 2 a difference of .1% is significant at the 95% level of confidence.

Certain results are clear. One can get 90% of the tags correct by just picking the most likely tag for each word. Improving the model to include bigrams of tags increases the accuracy to the 95% level, with the more theoretically pure \( P(w_i | t_i) \) performing better than \( P(t_i | w_i) \), contrary to the results in [1]. Furthermore the improvement is much larger than the .1% required for the 95% significance level. Improvement beyond this level is possible but it gets much harder. In particular, the improvement from the more sophisticated smoothing equation, Equation 26 is minimal, only .05%. This is not statistically significant. However, there is reason to believe that this understates the usefulness of this equation. In particular we believe that the very crude 23 is causing extra errors when combined with the improved smoothing. Equation 23 combined with the crudeness of our morphology component also limited the improvement shown in the last line of Figure 2. Also, we should really treat endings as tag “transformers,” something Equation 36 does not do. The combination of these three dominating factors caused frequent errors in known words, and thus the figure of 96.43% was obtained when we treated known words as morphologically primitive. This improvement is statistically significant. We believe that fixing these problems would add another tenth of a percent or two, but better performance beyond this will require more lexical information, as that used in [16].

However, the point of this paper was to clarify the basic equations behind tagging models, rather than improving the models themselves. We hope this paper encourages tag modelers to think about the mathematics which underly their models and to present their models in terms of the equations.

### Equations

<table>
<thead>
<tr>
<th>Equations</th>
<th>% Correct 471 Tags</th>
<th>% Correct 186 Tags</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 and 16</td>
<td>90.27</td>
<td>91.51</td>
</tr>
<tr>
<td>20, 16, and 23</td>
<td>94.09</td>
<td>95.04</td>
</tr>
<tr>
<td>19, 16, and 23</td>
<td>95.15</td>
<td>95.97</td>
</tr>
<tr>
<td>19, 26, and 23</td>
<td>96.02</td>
<td>96.45</td>
</tr>
<tr>
<td>36, 26, and 23</td>
<td>96.45</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2: Results obtained from the various models

### References


